Tropical Cyclone Structure

Introduction

To this point in the semester, we have only briefly touched upon the salient structural features of a tropical cyclone. We now describe these in greater detail. Included in this are structural aspects of both the primary (horizontal) and secondary (vertical) circulations and the axisymmetric and asymmetric structure of a tropical cyclone. We begin by introducing the basic structure of a tropical cyclone, using that as a launching point for exploring the tropical cyclone secondary circulation and asymmetric structure in greater detail.

Key Concepts

- What are the salient characteristics of the primary circulation of a tropical cyclone?
- What are the salient characteristics and dynamics of a tropical cyclone's secondary circulation?
- What is the typical structure of a tropical cyclone rain band?
- What are secondary eyewalls and what impact do they have on tropical cyclone intensity?

Tropical Cyclone Structure: Overview

Tropical cyclones, as areas of low pressure, are characterized by cyclonic tangential and inflowing radial winds. The cyclonic winds associated with a tropical cyclone can extend out to over 1000 km from its center in the lower troposphere; this radial extent decays with increasing height. Tropical cyclones are warm core features, meaning that their intensity (as measured by the magnitude of the cyclonic tangential wind) decreases with increasing height. A tropical cyclone is most intense just above the top of the boundary layer, where frictional dissipation is minimized, and weakest in the upper troposphere, where the winds become anticyclonic and evacuate mass outward. Radial inflow is typically maximized within the boundary layer with weaker inflow observed into the middle troposphere. The radial inflow rapidly decelerates upon reaching the eyewall of the tropical cyclone. The resultant convergence leads to ascending motion over a deep vertical layer within the eyewall. Compensatory descent for such strong ascent occurs in a concentrated manner within the eye and in a diffuse manner at radii larger than the radius of maximum winds.

The warm core structure of a tropical cyclone can be viewed as the hydrostatic response to a radially-constrained warm potential temperature anomaly near the center of the tropical cyclone. This warm anomaly primarily results from latent heat energy extracted from the underlying surface that is released in the upper troposphere by convective updrafts. A small but non-negligible contribution to this warm anomaly is also observed from subsidence warming within the eye.

In a planar view, a mature tropical cyclone is characterized by a nearly cloud-free region near its center, termed the eye. The minimum sea level pressure is found at the center of the eye. For weaker tropical cyclones without eye features, the minimum sea level pressure is typically found at the location of the greatest vertically-integrated potential temperature (i.e., where the warm anomaly associated with the tropical cyclone is strongest). The primary eyewall is found at the outermost radius of the eye. Here, intense convection and modestly strong updrafts (~5-10 m s^{-1}) are often found. The eyewall is often the location of the radius of maximum winds. On average, the eyewall and radius of maximum winds are typically found approximately 35 km from the center of the tropical cyclone; however, much smaller and much larger radii are often observed.

The eyewall region of a tropical cyclone is characterized by a local maximum in equivalent potential temperature. Isosurfaces of equivalent potential temperature are nearly vertical within the eyewall, implying that equivalent potential temperature is nearly constant with increasing height. Above the level of non-divergence, these surfaces flare radially outward. A local minimum in equivalent potential temperature is found in the middle to upper troposphere within the eye itself, reflecting drying associated with the aforementioned concentrated descent into the eye.

The eyewall and radius of maximum winds within a mature tropical cyclone slope outward with increasing height at an angle approaching 45°. This implies that the outward displacement of the eyewall in the upper troposphere (relative to its location at the surface) is approximately equivalent to its height above the sea surface. The physical reasoning behind this sloping structure lies with the conservation of angular momentum and the warm core structure of the tropical cyclone. As air parcels ascend within the eyewall, angular momentum is approximately conserved. Recall that angular momentum is both a function of the radius from the center of rotation (*r*) as well as the tangential wind speed (*v*), i.e.,

$$m = rv + \frac{fr^2}{2} \tag{1}$$

Since *v* decreases with increasing height, *r* must increase for *m* to remain constant.

A moat region, or region of predominantly stratiform precipitation, is found radially outward of the eyewall. Continuing radially outward from the moat region, mature tropical cyclones often possess secondary eyewalls. Most generically, secondary eyewalls form in response to the accumulation of heat energy, angular momentum, and vertical vorticity at some critical radius. The precise dynamics behind secondary eyewall formation remain unclear, however. Kossin and Sitkowski (2009) suggest that secondary eyewall formation is associated with high values of maximum potential intensity, small values of vertical wind shear, weak upper tropospheric zonal winds, a deep layer of underlying warm water, and high middle to upper tropospheric relative humidity. In other words, the factors that promote tropical cyclone intensification also tend to promote secondary eyewall formation.

The formation of a secondary eyewall temporarily halts the intensification of a tropical cyclone. The formation of a secondary eyewall effectively cuts off radial inflow into the inner eyewall. As the secondary eyewall matures and begins to contract, or move inward toward the center of the cyclone, compensating descent acts to erode the inner eyewall and clear out the moat region. After approximately 1-2 days, the inner eyewall has completely dissipated, leaving the tropical cyclone in a modestly weaker state. The larger eye and broadening of the tropical cyclone's wind field that result from this process, however, result in a stronger storm (as assessed by area-integrated kinetic energy). Reintensification is possible after the culmination of an eyewall replacement cycle assuming an otherwise favorable environment and the delayed formation of another secondary eyewall.

Beyond the secondary eyewall are found the rain bands of the tropical cyclone. Such rain bands can be characterized as primary, secondary, or distant. Distant rain bands are composed of deep, moist convection along confluence lines in the outer regions (e.g., radii > 200 km) of the tropical cyclone. Distant rain bands occur primarily where the environmental convective available potential energy (CAPE) is largest. Significant vertical motions (on the order of tens of meters per second in both upward and downward directions) and lightning activity are often found along distant rain bands, implying a predominantly convective nature to these features. Tornadic activity is possible along distant rain bands, particularly with those found in the right-front quadrant of landfalling tropical cyclones.

Secondary rain bands are typically found somewhat radially inward of a principal rain band. Often, the two intersect one another. Secondary rain bands are believed to be manifestations of vortex Rossby waves. They propagate cyclonically and outward at a rate of speed much less than the mean tangential flow of the tropical cyclone. Wave energy associated with a vortex Rossby wave accumulates at what is called a stagnation radius, whereby the strong tangential flow of the tropical cyclone shears and mixes the energy about the tropical cyclone. This shearing and mixing process is often referred to as the axisymmetrization process.

The principal rain band of a tropical cyclone lies predominantly within its inner core region (e.g., radii < 200 km). Principal rain bands tend to remain stationary in a storm-relative reference frame (i.e., they don't rotate around the cyclone to a significant extent). The precise dynamics behind their formation are unclear, however. A principal rain band is characterized by new convection on its upwind flank, mature convection in its core, and more stratiform-like precipitation on its downwind flank. Principal rain bands are typically found radially inward of a localized middle tropospheric jet, termed a secondary horizontal wind maximum. Convective activity within principal rain bands slopes radially outward, as with eyewall convection, but is generally constrained to within the lowest 8-10 km of the atmosphere.

Individual convective elements within principal rain bands are associated with both updraft and downdraft structures. Downdrafts are typically found radially inward of updrafts and result in a sharp horizontal edge to the region of precipitation associated with the rain band. Low entropy (or low equivalent potential temperature) air associated with such downdrafts can subsequently interact with the eyewall convection and act to reduce its vigor, as described in our earlier lecture on tropical cyclone intensity change. Updrafts act to transport high entropy (or high equivalent potential temperature) air upward within the rain band. Dynamically, updrafts act to tilt and subsequently vertically stretch horizontal vorticity found beneath the secondary horizontal wind maximum. Vertical advection of the newly-generated vertical vorticity acts to accumulate vertical vorticity within the middle troposphere, subsequently intensifying the secondary horizontal wind maximum.

In the remainder of this lecture, we endeavor to describe the secondary circulation of a tropical cyclone via the use of the Sawyer-Eliassen non-linear balance framework. We do so in order to better describe the evolution of a tropical cyclone in response to localized heat and momentum forcing.

Secondary Circulation: Sawyer-Eliassen Framework

As we have inferred numerous times previously throughout this class, the secondary circulation of a tropical cyclone is characterized by radial inflow at low levels, ascent near its center, and radial outflow near the tropopause. Colloquially, this is sometimes referred to as the "in-up-out" or axisymmetric circulation of the cyclone. This circulation is thermally direct in nature; i.e., as a tropical cyclone is associated with localized warmth at its core, the ascent occurs where it is warm. Compensating descent occurs at larger radii where it is relatively cooler. The exception to this, as discussed above, is within the eye, where locally warm air descends.

The Sawyer-Eliassen non-linear balance framework enables us to analytically describe the structure of this circulation as a function of the structure of the tropical cyclone and its environment. Furthermore, it also enables us to describe how this circulation evolves to the imposition of external heat (e.g., latent heat release) or momentum (e.g., trough interaction) forcing. In the following, we derive this equation from first principles of the atmosphere in a radial coordinate system.

Herein, the heat forcing is given by a prescribed heating Q. The momentum forcing is given by a prescribed momentum source (or sink) F. Both Q and F can take any desired form; however, in most analytical studies of the tropical cyclone secondary circulation, they take forms akin to such features within observed tropical cyclones. We will consider several structures for both Q and F when considering solutions to the Sawyer-Eliassen diagnostic equation.

Next, we introduce the governing equations. These are the governing equations represented in a two-dimensional (z,r) cylindrical coordinate system. In this regard, we view the secondary circulation as an axisymmetric feature. The governing equations are as follows:

(2) $$m^2 = r^3 \frac{\partial \phi}{\partial r}$$

(3) $$\frac{dm^2}{dt} = F$$

(4) $$\frac{\partial \Phi}{dz} = \frac{g}{\theta_0}\theta$$

(5) $$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0$$

(6) $$\frac{d\theta}{dt} = Q$$

(7) $$\Phi = \phi + \frac{f^2 r^2}{8}$$

(8) $$z = \left[1 - \left(\frac{\rho}{\rho_0}\right)^K\right]\frac{c_p \theta_0}{g}$$

(9) $$\frac{1}{r^3}\frac{\partial m^2}{\partial z}=\frac{g}{\theta}\frac{\partial \theta}{\partial r}$$

Equation (2) reflects gradient wind balance. Recalling that angular momentum is a function of the tangential wind *v*, equation (3) is thus a generic form of the tangential momentum equation. Equation (4) is the hydrostatic equation. Equation (5) is the flux form of the continuity equation. Recall that *u* denotes radial and not zonal wind. Equation (6) is the thermodynamic equation representing prescribed heating; i.e., this heating is diabatic in nature, and potential temperature does not change following the flow in the absence of this heating. Equation (7) is the definition of the geopotential. Equation (8) defines the pseudoheight vertical coordinate, used to simplify the mathematics of the system and interpretation thereof. The exponent *K* is equal to R_d/c_p. Finally, Equation (9) is the thermal wind relationship in this coordinate system, relating the vertical wind shear to horizontal temperature gradients. Except as described above, all variables have their standard meaning. Subscripts of *0* denote base-state values.

First, expand the total derivatives in (3) and (6) to obtain:

(10) $$\frac{\partial m^2}{\partial t}+u\frac{\partial m^2}{\partial r}+w\frac{\partial m^2}{\partial z}=F$$

(11) $$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial r}+w\frac{\partial \theta}{\partial z}=Q$$

Next, take the partial derivative of (10) with respect to *z* and multiply the result by $1/r^3$:

(12) $$\frac{\partial}{\partial t}\left(\frac{1}{r^3}\frac{\partial m^2}{\partial z}\right)+\frac{\partial}{\partial z}\left(u\frac{1}{r^3}\frac{\partial m^2}{\partial r}+w\frac{1}{r^3}\frac{\partial m^2}{\partial z}\right)=\frac{1}{r^3}\frac{\partial F}{\partial z}$$

Similarly, take the partial derivative of (11) with respect to *r* and multiply the result by g/θ_0:

(13) $$\frac{\partial}{\partial t}\left(\frac{g}{\theta_0}\frac{\partial \theta}{\partial r}\right)+\frac{\partial}{\partial r}\left(u\frac{g}{\theta_0}\frac{\partial \theta}{\partial r}+w\frac{g}{\theta_0}\frac{\partial \theta}{\partial z}\right)=\frac{g}{\theta_0}\frac{\partial Q}{\partial r}$$

Note that in obtaining (12) and (13), the partial derivatives with respect to time have been commuted with the partial derivatives with respect to *z* and *r*, respectively.

Before proceeding, it is useful to define several additional terms so as to enable the simplification of (12) and (13). These are as follows:

(14) $$N^2=\frac{g}{\theta_0}\frac{\partial \theta}{\partial z}$$

(15) $$B=-\frac{g}{\theta_0}\frac{\partial \theta}{\partial r}=-\frac{1}{r^3}\frac{\partial m^2}{\partial z}$$

(16) $$I = \frac{1}{r^3}\frac{\partial m^2}{\partial r} = \left(f + \frac{1}{r}\frac{\partial (rv)}{\partial r}\right)\left(f + \frac{2v}{r}\right)$$

Equation (14) defines static stability. Equation (15) defines baroclinicity. Equation (16) defines the inertial stability. Applying these definitions to (12) and (13) results in the following:

(17) $$\frac{\partial}{\partial t}(-B) + \frac{\partial}{\partial z}(Iu - Bw) = \frac{1}{r^3}\frac{\partial F}{\partial z}$$

(18) $$\frac{\partial}{\partial t}(-B) + \frac{\partial}{\partial r}(-Bu + N^2 w) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r}$$

Next, subtract (17) from (18) to obtain:

(19) $$\frac{\partial}{\partial r}(N^2 w - Bu) + \frac{\partial}{\partial z}(Bw - Iu) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r} - \frac{1}{r^3}\frac{\partial F}{\partial z}$$

Equation (19) describes the response in the zonal and vertical motion fields to imposed heat and/or momentum forcing. However, as there are two unlinked unknowns given by u and w, this equation is difficult to solve. To link these two variables and thus make solving the diagnostic equation simpler, the definition of the streamfunction is used. The definition of the streamfunction in this coordinate system is given by:

(20) $$u = -\frac{\partial \psi}{\partial z}, w = \frac{1}{r}\left(\frac{\partial (r\psi)}{\partial r}\right)$$

Substituting (20) into (19) results in the following:

(21) $$\frac{\partial}{\partial r}\left(N^2 \frac{1}{r}\frac{\partial (r\psi)}{\partial r} + B\frac{\partial \psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(B\frac{1}{r}\frac{\partial (r\psi)}{\partial r} + I\frac{\partial \psi}{\partial z}\right) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r} - \frac{1}{r^3}\frac{\partial F}{\partial z}$$

Equation (21) is the Sawyer-Eliassen non-linear secondary circulation diagnostic equation. It highlights the relationship between the specified heating Q, momentum forcing F, and the streamfunction ψ as modulated by coefficients representing static stability, inertial stability, and baroclinicity. The streamfunction attempts to restore the thermal wind balance that the specified heating and/or momentum forcing destroys. While thermal wind balance restoration is never truly achieved, the concepts of balance destruction and restoration nevertheless enable us to consider how radial and vertical motions (i.e., the strength of the secondary circulation) are impacted by prescribed heating and/or momentum forcing.

Solutions to the Sawyer-Eliassen diagnostic model are contained within the lecture materials. Herein, we focus upon describing these solutions and other salient characteristics related to the tropical cyclone secondary circulation. Both heat and cyclonic momentum sources drive localized vertical circulations. These vertical circulations can be characterized by the streamfunction, itself representative of the vertical and radial structures of vertical and radial motion.

A localized heat source forces ascent through the region of heating. This promotes convergence beneath and divergence above the heat source. Compensating descent occurs at larger radii. A localized cyclonic momentum source forces enhanced radial outflow through the region of cyclonic momentum. The totality of the resultant streamfunction response takes on a dipole structure in the vertical. Beneath the momentum source, ascent (descent) is promoted radially inward (outward) of the momentum source. The opposite is true above the momentum source: ascent (descent) is promoted radially outward (inward) of the momentum source. Enhanced radial inflow is promoted both above and below the cyclonic momentum source.

For both localized heat and cyclonic momentum sources, such motions are constrained in the vertical for weak inertial stability (small I) and constrained in the horizontal for strong inertial stability (large I). For a barotropic vortex (zero B), the forced vertical and radial motions are predominantly upright; for a baroclinic vortex (non-zero B), they are tilted somewhat in the vertical. The vertical extent of a given streamfunction response is restricted for large static stability (large N^2). For small static stability (small N^2), the vertical extent of a given streamfunction response is much less restricted.

Physically, localized heat and cyclonic momentum sources destroy stability, as assessed via thermal wind balance. In order for thermal wind balance to be restored, the effects of the heat or cyclonic momentum source must be mitigated through a balanced response in the cyclone's secondary circulation. For the heat source, this occurs via adiabatic cooling associated with the forced ascent. For the cyclonic momentum source, the radial outflow through the cyclonic momentum source acts to erode it and expel it from the tropical cyclone.

To first order, localized heat and cyclonic momentum sources within the upper troposphere act to enhance the secondary circulation of a tropical cyclone, particularly when each are located within the middle to upper troposphere. Strengthening the secondary circulation of a tropical cyclone enhances the rate of heat energy accumulation within the upper troposphere. Hydrostatically, this leads to the intensification of the primary circulation of the tropical cyclone via reduced surface pressure and, with gradient balance adjustment, enhanced surface winds. More specifically, a localized heat source leads to enhanced cyclonic tangential flow near and just inside of the radius of maximum heating. Weakened cyclonic tangential flow is found closer to the center of the tropical cyclone. A localized cyclonic momentum source results in enhanced cyclonic tangential flow radially inward of the radius at which the cyclonic momentum source is found.

The aforementioned impacts upon tropical cyclone intensity are greatest for relatively small, intense tropical cyclones. Weaker and/or larger tropical cyclones exhibit a positive yet somewhat weaker response to such forcing. Localized heating located at or inside of the radius of maximum winds (RMW) is most efficient at intensifying the tropical cyclone; expanding the horizontal extent of the heat source and/or moving it radially outward of the RMW is less efficient. Similarly, localized cyclonic momentum forcing located at or inside of the RMW is most efficient at intensifying the tropical cyclone; expanding its horizontal extent and/or moving it radially outward of the RMW is less efficient at doing so.

With respect to the impact of cyclonic momentum forcing upon the intensity of a tropical cyclone, it is important to keep in mind that such forcing is often associated with an upper tropospheric trough, which itself is often accompanied by vertical wind shear and dry middle and upper tropospheric air. Previously, we discussed the impacts upon the intensity of the tropical cyclone secondary circulation (as manifest via the Carnot heat engine approximation for a tropical cyclone) of dry air import and vertical wind shear (e.g., ventilation). Therefore, a tropical cyclone will only intensify if the positive impact of the cyclonic momentum source exceeds the magnitude of the negative impact of dry air import and vertical wind shear.

The radial structure of the primary circulation response to heat forcing provides a nice context from which the contraction of the RMW often observed with mature tropical cyclones can be explained. As the maximum acceleration in the tangential flow of the tropical cyclone is found radially inward of the RMW, the RMW will tend to move inward until equilibrium between the RMW and the radial location of the maximum response to the heat source is achieved. As the above implies, this context also makes clearer the process by which the primary eyewall is eroded by heat forcing found in association with a maturing secondary eyewall at larger radii.

References

- Houze, Jr., R. A., 2010: Clouds in tropical cyclones. *Mon. Wea. Rev.*, **138**, 293-344.
- Kossin, J. P., and M. Sitkowski, 2009: An objective model for identifying secondary eyewall formation in hurricanes. *Mon. Wea. Rev.*, **137**, 876-892.
- Pendergrass, A. G., and H. E. Willoughby, 2009: Diabatically induced secondary flows in tropical cyclones. Part I: quasi-steady forcing. *Mon. Wea. Rev.*, **137**, 805–821.
- Shapiro, L. J., and H. E. Willoughby, 1982: The response of balanced hurricanes to local sources of heat and momentum. *J. Atmos. Sci.*, **39**, 378-394.
- Sitkowski, M., J. P. Kossin, and C. M. Rozoff, 2011: Intensity and structure changes during hurricane eyewall replacement cycles. *Mon. Wea. Rev.*, **139**, 3829-3847.
- Vigh, J. L., and W. H. Schubert, 2009: Rapid development of the tropical cyclone warm core. *J. Atmos. Sci.*, **66**, 3335–3350.
- Willoughby, H. E., 1995: "Mature structure and evolution." *Global Perspectives on Tropical Cyclones*, R. L. Elsberry (ed.). World Meteorological Organization, Geneva, Switzerland, Report No. TCP-38. [Available online at http://www.meteo.physik.uni-muenchen.de/~roger/Tropical_Cyclones/global_perspectives_ch2.pdf].