Tropical Meteorology – Homework #1

Due Date: 18 February 2016

Introduction: In this and all future assignments, please note that clear, accurate, concise explanations will garner full credit, with accuracy and clarity being most important. Responses that are typed, hand-written, and/or a mixture of the two are all acceptable. You are welcome to work on and discuss the homework in groups; however, all responses must be your own. Please show all work for questions in which you are asked to perform a calculation.

An analog to the Kuo-Eliassen equation, often referred to as the Sawyer-Eliassen equation, describes the secondary circulation of a tropical cyclone. This secondary circulation reflects lower tropospheric radial inflow toward the center, ascent through the depth of the troposphere near the cyclone's center that is maximized in the middle troposphere, upper tropospheric radial outflow away from the center, and descent through the depth of the troposphere hundreds of kilometers away from the center of the cyclone.

A version of the Sawyer-Eliassen equation is given by:

$$\frac{\partial}{\partial r}(A^2 w + B^2 u) + \frac{\partial}{\partial z}(B^2 w - C^2 u) = \frac{g}{\theta}\frac{\partial H}{\partial r} - \frac{\partial(\xi M_F)}{\partial z}$$

In the above, u is the radial wind velocity and is positive for motion away from the center of the cyclone. The radial axis is thus also positive moving away from the center of the cyclone. ξ denotes the centrifugal stability and M_F denotes the momentum flux. The momentum flux is positive for the flux of cyclonic momentum. Here, we will consider the centrifugal stability to be a positive constant value. All other terms either have their typical meaning or are identical to the Kuo-Eliassen equation.

If we assume that the baroclinicity is negligibly small, this equation can be reduced to:

$$A^2 \frac{\partial w}{\partial r} - C^2 \frac{\partial u}{\partial z} = \frac{g}{\theta}\frac{\partial H}{\partial r} - \frac{\partial(\xi M_F)}{\partial z}$$

1. (19 pts) From this equation and the description given at the outset of this assignment, sketch and describe the secondary circulation of a tropical cyclone. Indicate location(s) where *each* of the partial derivatives on the *left-hand* side is positive or negative.

2. (19 pts) What are the vertical and radial distributions of momentum and heat, respectively, necessary to result in this secondary circulation? In other words, what must be the sign of each partial derivative on the *right-hand* side of this equation? Please consider each forcing term separately when answering this question.

3. (30 pts) Tropical cyclones are normally thought of as heat engines; in other words, just as with tropical overturning circulations, so too does heating fuel a tropical cyclone's winds. We now wish to consider the heating that is necessary to result in a surface pressure fall consistent with observed tropical cyclones.

    a. (7.5 pts) Consider a sounding taken over the tropical Atlantic Ocean. This sounding indicates that the 200 hPa height is 12,000 m and the 900 hPa height is 1,000 m. Using the hypsometric equation, please calculate the mean virtual temperature of the 900-200 hPa layer.

    b. (7.5 pts) Assume that, after some period of time, the surface pressure has fallen from an initial value of 1009 hPa to 960 hPa. Let the mean virtual temperature within the layer between the surface and 900 hPa be 25°C. Using the hypsometric equation, please determine the new 900 hPa height.

    c. (7.5 pts) Assume that the 200 hPa height remains constant at 12,000 m. Using the hypsometric equation along with your answer to (b), please determine the new mean virtual temperature of the 900-200 hPa layer.

    d. (7.5 pts) Much of the warming within the 900-200 hPa layer results from latent heat release in thunderstorms within the tropical cyclone inner core. Given your answers to (a) and (c), how long would it take for latent heat release at a rate of 2.5°C day^{-1} to lead to the observed pressure fall stated in (b)?

4. (32 pts) The conservation of absolute angular momentum within poleward-traveling air parcels in the upper tropospheric branch of the Hadley cell is in part responsible for the subtropical jet's existence. This jet is associated with some amount of kinetic energy that results from latent heat release in areas of deep, moist convection near the Equator. Thus, we assume a balance between latent heat release and kinetic energy generation.

    a. (4 pts) Consider a single thunderstorm within the tropics that results in 1.5 cm of rainfall in one day over the area cut out by a circle of 5 km diameter. What is the volume of water (in m) created by a single thunderstorm in one day?

b. (4 pts) Using the density of water and your answer to (a), what is the mass of water (in kg) created by a single thunderstorm in one day?

c. (4 pts) Using the latent heat of vaporization and your answer to (b), what is the latent heat (in J) released by a single thunderstorm in one day?

d. (4 pts) Climatologically, the subtropical jet is located near 25°N/25°S. We wish to calculate the surface area cut out between 25°S and 25°N. The surface area of this region can be expressed as $2\pi a^2(\sin \phi_n - \sin \phi_s)$, where ϕ_n is the northern latitude, ϕ_s is the southern latitude, and a is the radius of the Earth. Using this information, calculate the surface area (in m^2) between 25°S-25°N.

e. (4 pts) We now wish to find the mass of the atmospheric layer between 300-150 hPa, where the subtropical jet climatologically resides. Pressure can be defined as the force applied over an area and the force can be defined as the product of the mass and the acceleration. If the acceleration is that applied by gravity and the pressure is the pressure thickness of the 300-150 hPa layer, use your answer to (d) to calculate the total mass (in kg) of the 300-150 hPa layer between 25°S-25°N.

f. (4 pts) Assume that the mean wind speed between 25°S and 25°N is 20 m s^{-1}. Using this information and your answer to (e), calculate the total kinetic energy (in J) of the 300-150 hPa layer between 25°S-25°N.

g. (4 pts) Given a balance between latent heat release and kinetic energy generation, use your answers to (c) and (f) to calculate the number of thunderstorms per day that are required to generate the kinetic energy associated with the subtropical jet. In doing so, please assume that the total kinetic energy in (f) was generated over a period of three days as an air parcel traveled from the Equator to 25°S or 25°N in the poleward branch of the Hadley cell.

h. (4 pts) Using the equation for surface area in (d), please calculate the surface area (in m^2) between 10°S and 10°N, the latitudinal band in which most thunderstorms in the tropics are found. Using this, your answer to (g), and the thunderstorm diameter given in (a), find the fractional coverage of thunderstorms in the tropics.