<u>Tropical Meteorology – Homework #1</u>

Due Date: 15 February 2018

Introduction: In this and all future assignments, note that clear, accurate, concise explanations will garner full credit, with accuracy and clarity being most important. Responses that are typed, hand-written, and/or a mixture of the two are all acceptable. You are welcome to work on and discuss the homework in groups; however, *all responses must be your own*. Please show all work, including units where applicable, for questions in which you are asked to perform a calculation.

Learning Objectives: In this assignment, you will *apply* your understanding of the Hadley cell to generate new understanding about tropical cyclone structure. You will also quantitatively estimate the diabatic heating necessary to (a) produce a particular intensity change with a tropical cyclone and (b) generate a Hadley cell-like circulation to result in a subtropical jet of a particular intensity.

An analog to the Kuo-Eliassen equation that we invoked to describe the Hadley cell, often referred to as the Sawyer-Eliassen equation, describes the secondary circulation of a tropical cyclone. This secondary circulation reflects lower-tropospheric radial inflow toward the center, tropospheric-deep ascent near the cyclone's center that is maximized at mid-levels, upper-tropospheric radial outflow away from the center, and weak tropospheric-deep descent hundreds of kilometers away from the center.

A version of the Sawyer-Eliassen equation is given by:

$$\frac{\partial}{\partial r}(A^2 w + B^2 u) + \frac{\partial}{\partial z}(B^2 w - C^2 u) = \frac{g}{\theta}\frac{\partial H}{\partial r} - \frac{\partial(\xi F)}{\partial z}$$

Here, u is the radial wind (positive for motion away from the center). The radial axis is thus also positive directed away from the center of the cyclone.

ξ denotes the centrifugal stability:

$$\xi = f + 2\frac{v}{r}$$

where v is the tangential wind (positive for cyclonic rotation) and r is the distance from the center. For a cyclone, the centrifugal stability is positive-definite and is largest near the center, where v is large and r is small, and for more intense cyclones.

F denotes the momentum flux. For our purposes, this term can crudely be approximated as:

$$F \approx -\overline{u'\zeta'}$$

Here, overbars denote the azimuthal average (i.e., that taken at a constant radius around the entire cyclone) and primes denote departures from the azimuthal average. The right-hand side term is the eddy radial vorticity flux, or localized inward or outward transport of localized relative vorticity. The momentum flux is positive for inward ($u' < 0$) transport of cyclonic relative vorticity ($\zeta' > 0$) or outward ($u' > 0$) transport of anticyclonic relative vorticity ($\zeta' < 0$).

As for the Kuo-Eliassen equation, *A* is a measure of static stability, *B* is a measure of baroclinicity, *C* is a measure of inertial stability, and *H* is a heating term (positive for warming). If we assume that the baroclinicity is negligibly small, this equation can be reduced to:

$$A^2 \frac{\partial w}{\partial r} - C^2 \frac{\partial u}{\partial z} = \frac{g}{\theta} \frac{\partial H}{\partial r} - \frac{\partial (\xi F)}{\partial z}$$

Note that we will continue to assume that *A* and *C* are positive-definite. You are to use this equation and associated introduction to answer the first two questions below.

1. (19 pts) From this equation and the description given at the outset of this assignment, sketch and describe the secondary circulation of a tropical cyclone (note: only half of the tropical cyclone is necessary). Indicate location(s) where *each* of the partial derivatives on the *left-hand* side is positive or negative.

2. (19 pts) What are the vertical and radial distributions of heating and the momentum flux, respectively, necessary to result in this secondary circulation? In other words, what must be the sign of each partial derivative on the *right-hand* side of this equation? Consider each forcing term separately when answering this question.

3. (30 pts) Tropical cyclones are normally thought of as heat engines; in other words, just as with tropical overturning circulations, so too does heating fuel a tropical cyclone's winds. We now wish to consider the heating that is necessary to result in an approximate surface pressure fall – in as much as hydrostatic balance applies – that is consistent with observed tropical cyclones.

    a. (7.5 pts) Consider a sounding taken over the tropical north Atlantic Ocean. In this sounding, the 200 hPa and 900 hPa heights are 12,250 m and 900 m, respectively. Using the hypsometric equation, please calculate the mean virtual temperature of the 900-200 hPa layer for this sounding.

    b. (7.5 pts) Over some indeterminate time, the surface pressure falls from an initial value of 994 hPa to 960 hPa. If the mean virtual temperature in the layer between the surface (960 hPa) and 900 hPa is 25°C, using the hypsometric equation, please calculate the new 900 hPa height.

c. (7.5 pts) Assume that the 200 hPa height remains constant at 12,250 m. Using the hypsometric equation and your answer to (b), please calculate the new mean virtual temperature of the 900-200 hPa layer.

d. (7.5 pts) Much of the warming within the 900-200 hPa layer results from latent heat release in thunderstorms within the tropical cyclone inner core. Given your answers to (a) and (c), how long would it take for latent heat release at a rate of 2.4°C day^{-1} to lead to the observed pressure fall stated in (b)?

4. (32 pts) Absolute angular momentum conservation in poleward-traveling air parcels in the upper-tropospheric branch of the Hadley cell is partially responsible for the subtropical jet. The subtropical jet is associated with some amount of kinetic energy that results from latent heat release in areas of deep, moist convection near the Equator. Thus, we assume a balance between latent heat release and kinetic energy generation.

   a. (4 pts) Consider a single thunderstorm within the tropics that results in 1.75 cm of rainfall in 24 h over an area cut out by a circle of 4 km diameter. What is the volume of water (in m) created by this thunderstorm in 24 h?

   b. (4 pts) Using the density of water and your answer to (a), what is the mass of water (in kg) created by this thunderstorm in 24 h?

   c. (4 pts) Assume that the only phase changes occurring within this thunderstorm are associated with vapor condensing into liquid. Using the latent heat of vaporization and your answer to (b), what is the latent heat (in J) released by this thunderstorm in 24 h?

   d. (4 pts) Climatologically, the subtropical jet is located near 25°N/25°S. We wish to calculate the surface area cut out between 25°S and 25°N. The surface area of this region can be expressed as $2\pi a^2(\sin \phi_n - \sin \phi_s)$, where ϕ_n is the northern latitude, ϕ_s is the southern latitude, and a is the radius of the Earth. Using this information, calculate the surface area (in m^2) between 25°S-25°N.

   e. (4 pts) We now wish to find the mass of the atmospheric layer between 300-150 hPa, where the subtropical jet climatologically resides. Pressure can be defined as the force applied over an area, and the force can be defined as the product of the mass and the acceleration. If the acceleration is that applied by gravity and the pressure is the pressure thickness of the 300-150 hPa layer, use your answer to (d) to calculate the total mass (in kg) of the 300-150 hPa layer between 25°S-25°N.

f. (4 pts) Assume that the mean wind speed between 25°S and 25°N in the 300-150 hPa layer is 20 m s^{-1}. Using this information and your answer to (e), calculate the total kinetic energy (in J) contained in the 300-150 hPa layer between 25°S-25°N.

g. (4 pts) Given a balance between latent heat release and kinetic energy generation (i.e., one equals the other), use your answers to (c) and (f) to calculate the number of thunderstorms per day required to generate the kinetic energy of the subtropical jet. In doing so, please assume that the total kinetic energy in (f) was generated over a period of three days as an air parcel traveled from the Equator to 25°S or 25°N in the poleward branch of the Hadley cell.

h. (4 pts) Using the equation for surface area in (d), please calculate the surface area (in m^2) between 10°S and 10°N, the latitudinal band in which most thunderstorms in the tropics are found. Using this, your answer to (g), and the diameter given in (a), calculate the fractional coverage of thunderstorms in the tropics.