Synoptic Meteorology II: Contrasting the IPV and Quasi-Geostrophic Frameworks

28-30 April 2015

Readings: No formal readings.

Introduction

The quasi-geostrophic system of equations is a powerful system. As we demonstrated earlier in the semester, the quasi-geostrophic vorticity and height tendency equations may be used to describe the motion and evolution of upper tropospheric troughs and ridges. The quasi-geostrophic omega or Q-vector equations may be used to diagnose synoptic-scale vertical motion, while the latter may also be used to evaluate frontogenesis. The Peterssen-Sutcliffe development equation may be used to diagnose synoptic-scale surface cyclone development, motion, and evolution.

In recent weeks, we have introduced isentropic potential vorticity, including its conservation and mathematical formulation. We described the structure of upper tropospheric isentropic potential vorticity and surface potential temperature anomalies. We introduced the concept of isentropic potential vorticity inversion and stated how, given an appropriate balance condition, it can be used to obtain the thermal and kinematic fields associated with a given isentropic potential vorticity anomaly. We evaluated how diabatic heating and friction impact the three-dimensional distribution of isentropic potential vorticity.

To this point, however, what we have *not* done with isentropic potential vorticity is show how isentropic potential vorticity principles may be applied in order to describe the movement of the upper tropospheric pattern, synoptic-scale vertical motion, or synoptic-scale surface cyclone development, motion, and evolution. Indeed, as we will demonstrate in this lecture, isentropic potential vorticity principles may be used to describe each of these concepts, thus highlighting how isentropic potential vorticity "thinking" is a ***complement*** to quasi-geostrophic "thinking."

The Movement of the Upper Tropospheric Trough/Ridge Pattern

Let us first consider an atmosphere with no pre-existing relative vorticity ($\zeta = 0$) and no background flow. As a consequence, we can express the isentropic potential vorticity as:

$$P = -gf\frac{\partial \theta}{\partial p} \tag{1}$$

Since f increases in magnitude with increasing latitude, P is larger in magnitude toward the poles and smaller in magnitude toward the equator. This is depicted in Figure 1 below.

Figure 1. Horizontal distribution of isentropic potential vorticity P resulting exclusively from meridional variability in the Coriolis parameter f.

Let us now superimpose a series of alternating positive and negative upper tropospheric isentropic potential vorticity anomalies upon the distribution of P highlighted in Figure 1. The result of doing so along the latitude where $P = P$ is depicted in Figure 2.

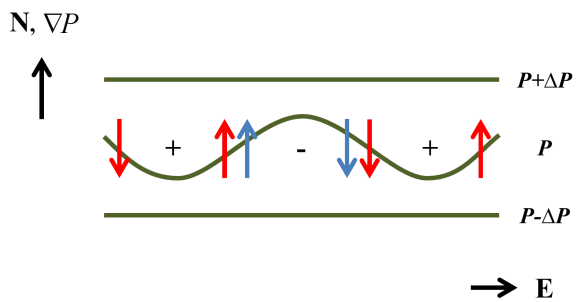

Figure 2. Horizontal distribution of isentropic potential vorticity P resulting from the superposition of the meridional variability in the Coriolis parameter f and a series of alternating positive (+) and negative (-) upper tropospheric isentropic potential vorticity anomalies. The flow associated with these anomalies is depicted in red and blue arrows, respectively.

By definition, positive upper tropospheric isentropic potential vorticity anomalies are associated with cyclonic flow. Likewise, negative upper tropospheric isentropic potential vorticity anomalies are associated with anticyclonic flow. These flows are depicted by arrows in Figure 2.

The advection of P by the alternating cyclonic and anticyclonic vortices results in positive P advection to the west of a positive isentropic potential vorticity anomaly and negative P advection to the west of a negative isentropic potential vorticity anomaly. This results in the

westward movement of the upper tropospheric trough/ridge pattern associated with the alternating positive and negative upper tropospheric isentropic potential vorticity anomalies!

In our lecture entitled "Potential Vorticity Inversion and Anomaly Structure," we demonstrated that the relative strength of the wind field induced by (or associated with) a given isentropic potential vorticity anomaly is directly proportional to the horizontal length scale of that anomaly,

$$U = \sigma^*_{ref} PL \tag{2}$$

where L is the horizontal length scale. Thus, isentropic potential vorticity anomalies of greater horizontal extent (larger L) are associated with stronger induced wind fields. Conversely, isentropic potential vorticity anomalies of lesser horizontal extent (larger L) are associated with weaker induced wind fields. Thus, the westward propagation described above is more rapid for larger-scale isentropic potential vorticity anomalies (e.g., longwaves) and less rapid for smaller-scale isentropic potential vorticity anomalies (e.g., shortwaves).

Finally, let us impose a background westerly flow upon our example. The combination of this westerly flow and the westward motion of the pattern that results from isentropic potential vorticity advection allows us to state the following:

- Longwave troughs retrogress to the west, against the large-scale flow, or move eastward at a relatively slow rate of speed.
- Shortwave troughs move to the east at a rate of speed that is equal to or somewhat less than that of the large-scale westerly flow.

These are the same conclusions that we made when we considered the geostrophic relative vorticity advection and planetary vorticity advection forcing terms contained within the quasi-geostrophic vorticity equation!

The Movement of Surface Cyclones and Anticyclones

The conceptual framework for understanding the motion of surface cyclones and anticyclones is very similar to that associated with the upper tropospheric trough/ridge pattern. Rather than considering the background meridional isentropic potential vorticity distribution, however, we consider the background meridional potential temperature distribution at the surface. Why? Remember: surface potential temperature anomalies are analogous to upper tropospheric isentropic potential vorticity anomalies!

On level terrain and neglecting diabatic processes, the background potential temperature distribution at the surface is largely determined by the meridional distribution of solar insolation.

Areas near the equator are warmer because they receive greater solar insolation, while areas near the poles are colder because they receive lesser solar insolation. The resultant background potential temperature distribution at the surface is depicted in Figure 3 below.

Figure 3. The horizontal distribution of potential temperature that results from the meridional variability in solar insolation.

Let us now superimpose a series of alternating warm and cold surface potential temperature anomalies upon the distribution of θ highlighted in Figure 3. The result of doing so along the latitude where $\theta = \theta$ is depicted in Figure 4.

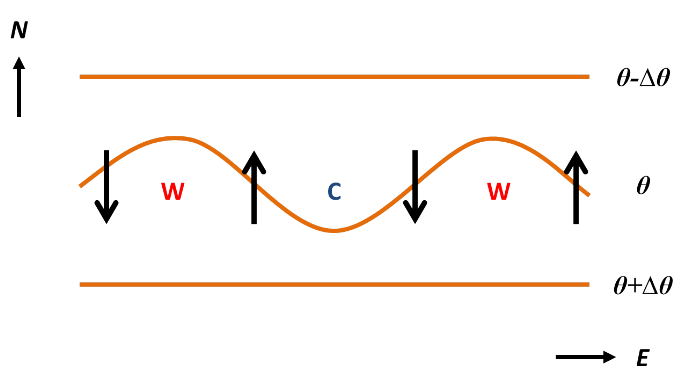

Figure 4. The horizontal distribution of potential temperature that results from the superposition of the distribution depicted in Figure 3 with a series of alternating warm (W) and cold (C) surface potential temperature anomalies. The flow associated with these anomalies is depicted by the black arrows.

When we examined the structure of surface potential temperature anomalies, we found that warm surface potential temperature anomalies are analogous to positive upper tropospheric isentropic potential vorticity anomalies and that cold surface potential temperature anomalies are analogous to negative upper tropospheric isentropic potential vorticity anomalies. Thus, warm surface potential temperature anomalies are associated with cyclonic flow and cold surface potential temperature anomalies are associated with anticyclonic flow. This is depicted by the arrows in Figure 4.

The advection of potential temperature by the induced cyclonic and anticyclonic vortices results in warm potential temperature advection east of warm surface potential temperature anomalies and cold potential temperature advection east of cold surface potential temperature anomalies. As a result, surface cyclones and anticyclones move ***eastward***. This is the same insight that we drew from the thermal advection term within the Peterssen-Sutcliffe development equation: surface cyclones move toward areas of lower tropospheric warm advection (e.g., toward areas of cyclone development) and surface anticyclones move toward areas of lower tropospheric cold advection (e.g., away from areas of cyclone development).

Diagnosing Vertical Motion with Upper Tropospheric IPV Anomalies

To begin, let us first consider an positive upper tropospheric isentropic potential vorticity anomaly embedded within westerly vertical wind shear. This is depicted in Figure 5 below. Note that similar arguments to the ones that we will make below can be made for upper tropospheric isentropic potential vorticity anomalies of either sign in any vertically-sheared environment.

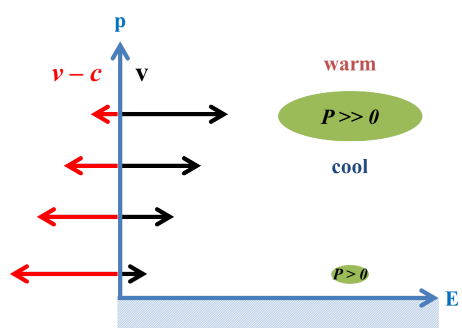

Figure 5. Vertical cross-section depicting an positive upper tropospheric isentropic potential vorticity anomaly (large green oval), its induced lower tropospheric positive isentropic potential vorticity anomaly (small green oval), the synoptic-scale horizontal wind field (black arrows), the storm-relative horizontal wind field (red arrows), and the relative potential temperature above and below the upper tropospheric isentropic potential vorticity anomaly.

Let us consider a reference frame that moves with the positive upper tropospheric isentropic potential vorticity anomaly. We will call this the storm-relative reference frame. Let us assume that the positive upper tropospheric isentropic potential vorticity anomaly moves eastward with the westerly *upper tropospheric flow*. If we subtract this storm motion from the wind field at each level depicted in Figure 5, we obtain a storm-relative easterly flow that decreases with height. This is depicted by the red arrows on the left-hand side of Figure 5.

Alone, the isentropic potential vorticity framework does not contain a diagnostic equation for vertical motion (ω). Thus, an equation from outside of this framework that enables us to diagnose vertical motion must be considered. Recall that the quasi-geostrophic omega equation is derived from a combination of the quasi-geostrophic vorticity and quasi-geostrophic thermodynamic equations, each of which contains one term involving ω. When diagnosing vertical motion in the isentropic potential vorticity framework, we can "borrow" *either* the quasi-geostrophic vorticity *or* the quasi-geostrophic thermodynamic equation to complete the analysis. Let us consider both options in the context of our aforementioned positive upper tropospheric isentropic potential vorticity anomaly.

Vorticity Perspective

Per the "action at a distance" principle, we know that the positive upper tropospheric isentropic potential vorticity anomaly induces a weak cyclonic vortex beneath it at the surface, also as depicted in Figure 5.

The quasi-geostrophic vorticity equation on an isobaric surface, neglecting friction, is given by:

$$\frac{\partial \zeta_g}{\partial t} = -\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right) + f_0 \frac{\partial \omega}{\partial p} \tag{3}$$

where in (3), we combined the geostrophic relative vorticity and planetary vorticity advection terms into a single advection term.

Let us state that in the storm-relative reference frame, ζ_g does not change with time. This approximation allows us to write (3) as:

$$-\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right) = -f_0 \frac{\partial \omega}{\partial p} \tag{4}$$

where it should be emphasized that the advection term in this reference frame is the *storm-relative advection*.

The easterly storm-relative flow at the surface wants to advect the weak surface cyclonic vortex westward. In the storm-relative reference frame, this imparts storm-relative lower tropospheric cyclonic geostrophic absolute vorticity advection ($-\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right) > 0$) to the west and storm-relative lower tropospheric anticyclonic geostrophic absolute vorticity advection ($-\vec{\mathbf{v}}_g \cdot \nabla(\zeta_g + f) < 0$) to the east of the weak surface cyclonic vortex.

From (4), where $f_0 > 0$, this results in $\partial\omega/\partial p < 0$ to the west and $\partial\omega/\partial p > 0$ to the east of the weak surface cyclonic vortex. For $\partial p < 0$, this means that $\partial\omega > 0$ to the west and $\partial\omega < 0$ to the east of the weak surface cyclonic vortex. If we take $\omega = 0$ at the ground, then $\partial\omega = \omega_{mid} - \omega_{sfc} = \omega_{mid}$, where the subscript of *mid* indicates the middle troposphere. Thus, we find that $\omega_{mid} > 0$ to the west, indicating descent, and $\omega_{mid} < 0$ to the east, indicating ascent.

For a sanity check, let us compare this to insight garnered from the quasi-geostrophic omega equation. The positive IPV anomaly and background westerly wind are strongest in the upper troposphere. As a positive IPV anomaly is associated with cyclonic absolute vorticity, this results in cyclonic (geostrophic) absolute vorticity advection that increases with height to the east and anticyclonic (geostrophic) absolute vorticity advection that increases with height to the west. These are associated with middle-tropospheric descent and ascent, respectively, as stated above.

Thermodynamic Perspective

On an isobaric surface, neglecting diabatic heating, the quasi-geostrophic thermodynamic equation is given by:

$$\frac{\partial T}{\partial t} + \vec{\mathbf{v}}_g \cdot \nabla T - S_p \omega = 0 \tag{5}$$

In (5), we note that the temperature on an isobaric surface is directly analogous to the potential temperature due to Poisson's relationship. S_p is a measure of the static stability and is generally positive (potential temperature increasing with increasing height). Let us state that in the storm-relative reference frame, T does not change with time. This allows us to re-write (5):

$$-\vec{\mathbf{v}}_g \cdot \nabla T = -S_p \omega \tag{6}$$

where it should again be emphasized that the advection term in this reference frame is the *storm-relative advection*.

When we described the vertical structure of positive upper tropospheric isentropic potential vorticity anomalies, we noted that the potential temperature below the anomaly in the middle to upper troposphere is relatively cold. This is also noted on Figure 5. In the storm-relative reference frame, the weak easterly storm-relative flow beneath the positive isentropic potential vorticity anomaly results in cold storm-relative temperature advection ($-\vec{\mathbf{v}}_g \cdot \nabla T < 0$) west of the anomaly and warm storm-relative temperature advection ($-\vec{\mathbf{v}}_g \cdot \nabla T > 0$) east of the anomaly.

From (6), for $S_p > 0$, warm temperature advection results in $\omega < 0$, signifying ascent. Conversely, cold temperature advection results in $\omega > 0$, signifying descent.

The sanity check upon this utilizing the quasi-geostrophic omega equation is not as straightforward as it was in the vorticity-based perspective above. Consider, for instance, middle tropospheric potential temperature advection in this framework. We know that potential temperature is relatively low (cold) beneath the upper tropospheric positive isentropic potential vorticity anomaly. Advection of this by the background westerly wind would lead to cold advection – and thus descent – east and warm advection – and thus ascent – west of the anomaly. This is the opposite of the insight we just gained and, likely, from what we expect.

Recall, however, that the positive isentropic potential vorticity anomaly is embedded within westerly vertical wind shear. From thermal wind, this is associated with a north-south layer-mean temperature gradient, with colder temperatures to the north and warmer temperatures to the south. The cyclonic rotational flow induced in the middle troposphere by the positive isentropic potential vorticity anomaly thus results in warm advection to the east and cold advection to the west. In the quasi-geostrophic omega equation, these are associated with ascent and descent, respectively. While both advection of and by the induced anomaly typically occur, it is the latter (discussed in this paragraph) that typically dominates, thereby resulting in agreement between the isentropic potential vorticity-based and quasi-geostrophic omega-based interpretations.

Diagnosing Vertical Motion with Surface Potential Temperature Anomalies

Let us consider a warm surface potential temperature anomaly embedded within westerly vertical wind shear, with no flow at the surface and increasingly strong westerly flow aloft. This is depicted in Figure 6 below. Again per the thermal wind relationship, westerly vertical wind shear implies the presence of a north-south layer-mean temperature (or, equivalently on an isobaric surface, potential temperature) gradient with warm air to the south in the Northern Hemisphere. We consider a reference frame that "moves" with the stationary surface wind, such that the storm-relative wind is identical to the full wind.

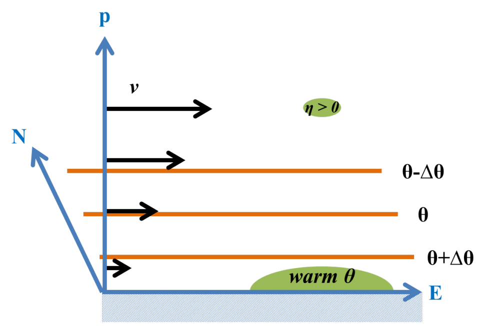

Figure 6. Three-dimensional cross-section depicting a surface warm potential temperature anomaly (large green oval), its induced upper tropospheric cyclonic vortex (small green oval), the synoptic-scale horizontal wind field (black arrows), and the meridional potential temperature gradient that is in thermal wind balance with the westerly vertical wind shear (orange lines).

Vorticity Perspective

Per the "action at a distance" principle, the warm surface potential temperature anomaly induces a weak cyclonic vortex in the upper troposphere. The westerly flow aloft acts to advect this induced vortex eastward. This results in cyclonic absolute vorticity advection to the east and anticyclonic absolute vorticity advection to the west. From (4), again for $f_0 > 0$, this results in $\partial\omega/\partial p < 0$ to the east and $\partial\omega/\partial p > 0$ to the west of the weak upper tropospheric cyclone. For $\partial p < 0$, this means that $\partial\omega > 0$ to the east and $\partial\omega < 0$ to the west of the weak upper tropospheric cyclone. If we take $\omega = 0$ at the tropopause, then $\partial\omega = \omega_{trop} - \omega_{mid}$. Thus, we find that $\omega_{mid} < 0$ to the east, indicating ascent, and $\omega_{mid} > 0$ to the west, indicating descent.

The sanity check upon this result utilizing the quasi-geostrophic omega equation is not straightforward. The magnitude of the warm anomaly – and induced cyclonic vortex – is greatest at the surface and decreases with increasing height. However, the background westerly wind increases in magnitude with increasing height. How, then, does the magnitude of vorticity advection change with increasing height? Generally speaking, it will increase as you move upward – little to no advection at the surface given little to no wind versus larger advection aloft with faster wind (albeit a weaker absolute vorticity anomaly to advect). For the scenario considered in Figure 6, this leads to cyclonic absolute vorticity advection increasing with height – and thus ascent – to the east and anticyclonic absolute vorticity advection increasing with height – and thus descent – to the west, in agreement with the above interpretation.

Thermodynamic Perspective

For this example, the interpretation from the thermodynamic perspective is relatively straightforward. The weak upper tropospheric cyclone is associated with southerly flow to its east and northerly flow to its west. As noted above, from the thermal wind relationship, westerly vertical wind shear is associated with a north-south temperature gradient with warm air to the south in the Northern Hemisphere.

Thus, there is warm air advection to the east of the weak induced upper tropospheric cyclone and cold air advection to the west of the weak induced upper tropospheric cyclone. From (6), for $S_p > 0$, warm temperature advection results in $\omega < 0$, signifying ascent. Conversely, cold temperature advection results in $\omega > 0$, signifying descent. This is in agreement with the interpretation from both the quasi-geostrophic omega equation – warm and cold advection in the middle troposphere resulting in ascent and descent, respectively – and the vorticity perspective illustrated above.