Synoptic Meteorology II: The Effects of Diabatic Heating and Friction Upon Isentropic Potential Vorticity

21-23 April 2015

Readings: Sections 4.3.1 and 4.3.2 of *Midlatitude Synoptic Meteorology*

Mathematical Formulation

In our derivation of the isentropic potential vorticity (IPV) P, we stated that the vorticity equation applicable on isentropic surfaces (neglecting friction and diabatic heating) takes the form:

$$\frac{\partial \zeta}{\partial t} + \vec{\mathbf{v}} \cdot \nabla_\theta (\zeta + f) + (\zeta + f)\nabla_\theta \cdot \vec{\mathbf{v}} = 0 \tag{1}$$

If we substitute $\eta = \zeta + f$, noting that f is constant with respect to both t and θ, and add in the effects of friction and diabatic heating, we obtain:

$$\frac{\partial \eta}{\partial t} + \vec{\mathbf{v}} \cdot \nabla_\theta \eta + \dot{\theta}\frac{\partial \eta}{\partial \theta} + \eta \nabla_\theta \cdot \vec{\mathbf{v}} = \hat{\mathbf{k}} \cdot \frac{\partial \vec{\mathbf{v}}}{\partial \theta} \times \nabla_\theta \dot{\theta} + \hat{\mathbf{k}} \cdot \nabla_\theta \times \vec{\mathbf{F}} \tag{2}$$

Also from that derivation, we stated that the continuity equation applicable on isentropic surfaces (neglecting diabatic heating) takes the form:

$$\frac{\partial}{\partial t}\left(\frac{\partial \theta}{\partial p}\right) - \frac{\partial \theta}{\partial p}\nabla_\theta \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \cdot \nabla_\theta \left(\frac{\partial \theta}{\partial p}\right) = 0 \tag{3}$$

If we no longer neglect diabatic heating, (3) becomes:

$$\frac{\partial}{\partial t}\left(\frac{\partial \theta}{\partial p}\right) - \frac{\partial \theta}{\partial p}\nabla_\theta \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \cdot \nabla_\theta \left(\frac{\partial \theta}{\partial p}\right) = \frac{\partial \theta}{\partial p}\frac{\partial \dot{\theta}}{\partial t} - \dot{\theta}\frac{\partial}{\partial \theta}\left(\frac{\partial \theta}{\partial p}\right) \tag{4}$$

Both (2) and (4) contain terms related to the diabatic heating rate $\dot{\theta}$, while (2) contains a term related to frictional processes.

We now wish to obtain an expression for the total derivative of P. Multiplying (2) by $-g\partial\theta/\partial p$ and adding to it (4) multiplied by $-g\eta$ allows us to eliminate the divergence term ($\nabla_\theta \cdot \vec{\mathbf{v}}$) found in both equations. If do so, make use of the chain rule to combine terms, and apply the definition of P to simplify the resultant equation, we obtain:

$$\frac{DP}{Dt} = -g\frac{\partial\theta}{\partial p}\hat{\mathbf{k}}\cdot\left(\frac{\partial\vec{\mathbf{v}}}{\partial\theta}\times\nabla_\theta\dot{\theta}\right) - g\eta\frac{\partial\theta}{\partial p}\frac{\partial\dot{\theta}}{\partial\theta} - g\frac{\partial\theta}{\partial p}\hat{\mathbf{k}}\cdot\left(\nabla_\theta\times\vec{\mathbf{F}}\right) \tag{5}$$

Equation (5) describes the *non-conservation* of isentropic potential vorticity as a function of diabatic heating – or, more specifically, vertical and horizontal gradients thereof – and friction.

The first forcing term on the right-hand side of (5) is known as the *shear diabatic* term because it is related to the cross-product of the vertical isentropic wind shear and the horizontal gradient of diabatic heating on an isentropic surface. The second forcing term on the right-hand side of (5) is known as the *vertical diabatic* term because it is related to the vertical structure of diabatic heating. The last forcing term on the right-hand side of (5) is simply the *friction* term.

If we combine the two diabatic heating terms into one by making use of the definition of the three-dimensional vorticity vector on an isentropic surface, we obtain:

$$\frac{DP}{Dt} = -g\frac{\partial\theta}{\partial p}\left(\nabla_3\times\vec{\mathbf{v}} + f\hat{\mathbf{k}}\right)\cdot\nabla_3\dot{\theta} - g\frac{\partial\theta}{\partial p}\hat{\mathbf{k}}\cdot\left(\nabla_\theta\times\vec{\mathbf{F}}\right) \tag{6}$$

In (6), subscripts of 3 indicate that the gradient applies in three dimensions (x, y, θ) on an isentropic surface. This equation makes clear that the isentropic potential vorticity changes following the motion based upon the orientation of the three-dimensional absolute vorticity vector with respect to the three-dimensional gradient of diabatic heating.

Interpretation of Diabatic Heating and Frictional Impacts upon IPV

We now wish to examine how each of the three forcing terms of (5) contributes to IPV non-conservation. Put differently, we now wish to examine how each of the three forcing terms of (5) contributes to local changes in IPV.

Vertical Diabatic Term

In isolation, the contribution of the vertical diabatic term to changes in IPV can be expressed as:

$$\frac{DP}{Dt} = -g\eta\frac{\partial\theta}{\partial p}\frac{\partial\dot{\theta}}{\partial\theta} = P\frac{\partial\dot{\theta}}{\partial\theta} \tag{7}$$

From (7), it is apparent that we are interested in evaluating $\partial\dot{\theta}/\partial\theta$. To illustrate doing so, let us consider the case where there is a middle-tropospheric maximum in diabatic warming ($\dot{\theta} > 0$). This is depicted in Figure 1 below. We note that diabatic warming is typically associated with latent heat release due to condensation. This tends to be maximized in the middle troposphere as that is where the best overlap between ascent and water vapor content occurs; water vapor content is small even when saturated in the upper troposphere, while ascent is relatively weak in the lower troposphere.

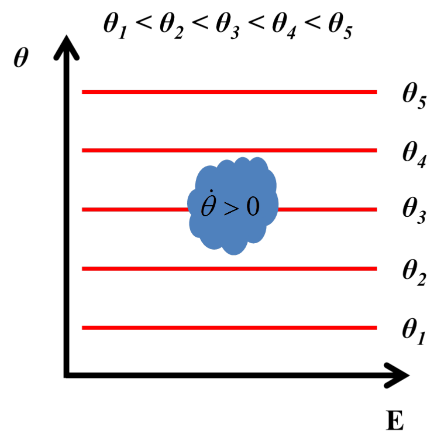

Figure 1. East-west vertical cross-section of isentropes (red lines) in the presence of a middle tropospheric maximum in diabatic warming (blue cloud).

Since θ increases as you go up (e.g., with height), $\partial\theta$ is positive. Below the level of the maximum in $\dot{\theta}$, $\dot{\theta}$ increases with height ($\partial\dot{\theta} > 0$). With both numerator and denominator positive, presuming that *P* is positive, *DP/Dt* is positive, signifying that the isentropic potential vorticity increases in the lower troposphere following the motion.

What does this imply about the role of diabatic warming in surface cyclogenesis? Middle tropospheric diabatic warming occurring above a surface cyclone increases both the *P* and η of the surface cyclone. Therefore, the effect of diabatic warming upon surface cyclogenesis is the same as that in the Petterssen-Sutcliffe development framework: it acts to enhance surface cyclogenesis!

Conversely, above the level of the maximum in $\dot{\theta}$, $\dot{\theta}$ decreases with height ($\partial\dot{\theta} < 0$). With the numerator negative and denominator positive, again presuming that *P* is positive, *DP/Dt* is negative. This signifies that the isentropic potential vorticity decreases in the upper troposphere following the motion.

We can confirm this conceptually by considering how the diabatic warming influences the components of the isentropic potential vorticity. We start with the static stability component of *P*, $-\partial\theta/\partial p$. Since θ generally increases with height, diabatic warming forces isentropes downward. This is depicted graphically in Figure 2.

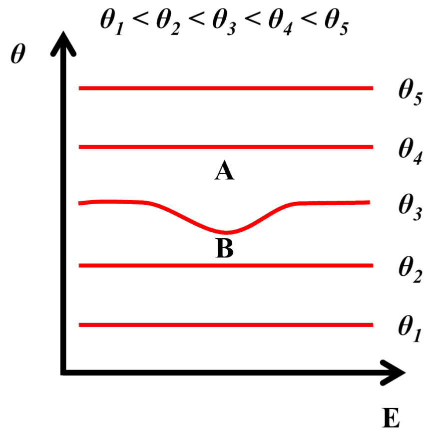

Figure 2. As in Figure 1, except depicting isentropes at a later time post-diabatic warming.

At point A, above the level of the diabatic warming maximum, $-\partial\theta/\partial p$ becomes smaller as diabatic warming pushes the isentropes further apart in the vertical. At point B, below the level of the diabatic warming maximum, $-\partial\theta/\partial p$ becomes larger as diabatic warming pushes the isentropes closer together in the vertical. Thus, from this term alone, P increases in the lower troposphere and decreases in the upper troposphere.

The impacts of diabatic heating upon the absolute vorticity η can be examined using two complementary perspectives…

1) *Quasi-geostrophic perspective*: From the quasi-geostrophic omega equation, $\dot{\theta}$ is proportional to $-\omega$. Thus, diabatic warming results in ascent ($\omega < 0$) that is maximized at the level at which the diabatic warming is maximized. Assuming that $\omega \approx 0$ at the surface and tropopause, with $\partial p < 0$, $\partial\omega/\partial p$ is positive below and negative above the level of the maximum in diabatic heating. From the quasi-geostrophic vorticity equation, the local rate of change of geostrophic relative vorticity – a major contributor to the absolute vorticity – is proportional to $\partial\omega/\partial p$. Thus, η and, by extension, P increases in the lower troposphere and decreases in the upper troposphere.

2) *Thickness perspective*: Diabatic warming increases the potential temperature of a vertical layer, increasing its thickness. This forces pressure surfaces downward below and upward above the level of maximum diabatic warming, creating locally low pressure in the lower troposphere and locally higher pressure in the upper troposphere. (As an aside, this can be confirmed by examination of the quasi-geostrophic height tendency equation.) From gradient wind balance, we know that lows are associated with convergence and highs are associated with divergence. The vorticity equation valid on an isentropic surface, as given by equation (6) of our introductory IPV lecture, states that the local rate of change of relative vorticity – a major contributor to the absolute vorticity – is a function of both η and the divergence. In the presence of cyclonic η, convergence acts to make η more

cyclonic while divergence acts to make η more anticyclonic. Thus, P increases in the lower troposphere and decreases in the upper troposphere.

The above arguments could be repeated for diabatic cooling, with the result being the opposite: diabatic cooling contributes to increasing P above and decreasing P below the height at which the diabatic cooling is maximized. We will discuss a couple of examples for which diabatic cooling may contribute to increasing P later in this lecture.

Shear Diabatic Term

In isolation, the contribution of the shear diabatic term to changes in IPV can be expressed as:

$$\frac{DP}{Dt} = -g\frac{\partial\theta}{\partial p}\hat{\mathbf{k}}\cdot\left(\frac{\partial\vec{\mathbf{v}}}{\partial\theta}\times\nabla_\theta\dot{\theta}\right) \tag{8}$$

If we expand (8) into its components, we obtain:

$$\frac{DP}{Dt} = -g\frac{\partial\theta}{\partial p}\left(\frac{\partial u}{\partial\theta}\frac{\partial\dot{\theta}}{\partial y} - \frac{\partial v}{\partial\theta}\frac{\partial\dot{\theta}}{\partial x}\right) \tag{9}$$

To interpret this, let us consider the case where there is southwesterly vertical isentropic wind shear. In addition, let us presume that there exists a middle tropospheric diabatic warming maximum to the northwest of a surface cyclone. These are depicted in Figure 3 below.

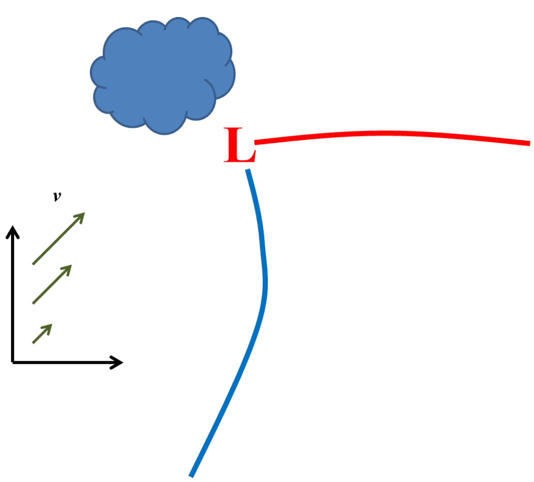

Figure 3. Depiction of a surface cyclone and its warm (red line) and cold (blue line) fronts located to the southeast of a middle tropospheric diabatic warming maximum (blue cloud). The vertical isentropic wind shear is denoted by the inset at left.

For southwesterly shear, as both u and v become more positive (westerly, southerly) as you go upward, both $\partial u/\partial\theta$ and $\partial v/\partial\theta$ are positive. Since the shear is purely southwesterly (i.e., $u = v$ at all heights), $\partial u/\partial\theta = \partial v/\partial\theta$ in this example. Let us also presume that the atmosphere is statically stable, such that $-\partial\theta/\partial p > 0$. Thus, given the sign convention on $\partial u/\partial\theta$ and $\partial v/\partial\theta$ for this case, $DP/Dt > 0$ for $\partial\dot{\theta}/\partial y > 0$ and $\partial\dot{\theta}/\partial x < 0$. Conversely, $DP/Dt < 0$ for $\partial\dot{\theta}/\partial y < 0$ and $\partial\dot{\theta}/\partial x > 0$.

Since the positive x-axis is directed toward the east, $\partial\dot{\theta}/\partial x$ is negative to the east and positive to the west of the diabatic warming maximum. Likewise, since the positive y-axis is directed toward the north, $\partial\dot{\theta}/\partial y$ is negative to the north and positive to the south of the diabatic warming maximum. For simplicity, we will assume that the magnitudes of $\partial\dot{\theta}/\partial x$ and $\partial\dot{\theta}/\partial y$ are equivalent everywhere. Thus, ***for the case of southwesterly isentropic vertical wind shear only,*** $DP/Dt < 0$ in the middle troposphere to the northwest of the diabatic warming maximum and $DP/Dt > 0$ in the middle troposphere to the southeast of the diabatic warming maximum.

Note that the effect of diabatic heating upon the non-conservation of P for different isentropic vertical wind shears can be evaluated in a similar fashion to the above, taking care to correctly evaluate the sign and magnitude of both $\partial u/\partial\theta$ and $\partial v/\partial\theta$.

Applications of Diabatic Heating and its Influences upon IPV

Via the invertibility principle, diabatically changing the isentropic potential vorticity over some area results in changes to its underlying thermal and kinematic fields. This can influence the direction and/or magnitude of the vertical isentropic wind shear and, by application of the thermal wind relationship, the magnitude of the horizontal temperature gradient over that layer.

This is particularly important for "tropical transition," or the transformation of an extratropical, mid-latitude synoptic-scale cyclone into a tropical cyclone. In the context of the vertical diabatic term, a middle tropospheric maximum of diabatic warming decreases P in the upper troposphere and increases P in the lower troposphere. This decreases the magnitude of the wind speed aloft while increasing it toward the surface, counteracting the typical increase in wind speed with increasing height in the mid-latitudes. This reduces the vertical wind shear in the vicinity of the diabatic warming maximum and, from thermal wind balance, the magnitude of the horizontal temperature gradient within the troposphere. Reduced vertical wind shear and weakened baroclinicity are necessary (but not sufficient) conditions for tropical cyclone development to occur. Thus, the development of deep, moist convection atop or just upshear of a mid-latitude synoptic-scale cyclone is a hallmark of the "tropical transition" process due to its effects on the vertical wind shear felt by the cyclone and the baroclinicity found across the cyclone.

The non-conservation of isentropic potential vorticity often plays an important role in the development of both mid-latitude and tropical cyclones, as we will discuss in more detail in this and later lectures. However, the non-conservation of P is not always well-forecasted or represented by numerical weather prediction models. Why not? Diabatic heating is difficult to represent accurately within a forecast model: it involves numerous complex physical processes that we don't know as much about as we'd like, it is computationally expensive for the forecast model to resolve it, and we don't have direct quantitative observations of it to help address these issues. As a result, errors in representing diabatic heating that are inevitably present within model forecasts can and often do lead to errors in the rest of the forecast – and, for some systems, such errors can be quite large and meaningful!

A Cautionary Note on the Physical Interpretation of Diabatic Heating's Effects on IPV

By use of the divergence (or Gauss' theorem), it can be shown that:

$$\int_V \frac{\partial(\sigma P)}{\partial t} dV = 0 \tag{10}$$

In (10), σ is related to the static stability. This equation states that the volume-integrated change of P with respect to time is zero. Presuming that an isentropic potential vorticity anomaly – or, more accurately, the effects of diabatic heating and friction upon that anomaly – are isolated from the external environment, the volume-integrated P **does not change** due to diabatic heating or friction.

Instead, P is rearranged both horizontally and vertically within the volume to effect local changes in P. Let us view this in the context of our middle tropospheric maximum of diabatic warming illustrated above in Figure 1. In this example, diabatic warming increases P in the lower troposphere and decreases P in the upper troposphere. Rather than actually creating or destroying P, however, the diabatic heating rearranges P. Since P typically increases with height, this can be viewed in the context of bringing higher P downward toward the surface and lower P upward toward the tropopause. Similar arguments can be made for diabatic cooling and/or in the horizontal (e.g., for the shear diabatic term).

Therefore, it is not appropriate to view the effects of diabatic heating (and friction) upon the non-conservation of P in the context of the creation or destruction of P – rather, one should view them in the context of the horizontal and vertical rearrangement of P.

Frictional Term

In isolation, the contribution of the frictional term to changes in IPV can be expressed as:

$$\frac{DP}{Dt} = -g\frac{\partial\theta}{\partial p}\hat{\mathbf{k}}\cdot\left(\nabla_{\theta}\times\vec{\mathbf{F}}\right) \tag{11}$$

As in the quasi-geostrophic system, friction acts as a *brake* on the intensity of boundary layer isentropic potential vorticity anomalies, whether cyclonic or anticyclonic in nature.

<u>The Role of Diabatic Heating and Friction Upon the Development and Decay of Upper Tropospheric Cyclonic and Anticyclonic IPV Anomalies</u>

In a general sense, we have described how diabatic heating and friction impact the isentropic potential vorticity. However, it is useful to further examine how diabatic heating and friction impact IPV by doing so in the context of the development and decay of middle- to upper-tropospheric cyclonic and anticyclonic IPV anomalies.

Decay of an Upper Tropospheric Positive IPV Anomaly

Let us first consider the decay of an upper tropospheric positive (cyclonic) isentropic potential vorticity anomaly. The magnitude of this anomaly is maximized at the height of the anomaly but, due to the "action at a distance" principle we discussed last week, it induces a positive IPV anomaly downward toward the surface.

In last week's lecture, when we related surface potential temperature anomalies to upper tropospheric IPV anomalies, we introduced the concept of thermal vorticity. Thermal vorticity relates the change of the geostrophic relative vorticity with height to the Laplacian of the potential temperature field, as expressed by:

$$f_0\frac{\partial\zeta_g}{\partial p} = -h\nabla^2\theta \tag{12}$$

Because the positive IPV anomaly and its associated cyclonic rotation increase in intensity with height ($\partial\zeta_g > 0$, $\partial p < 0$), the left-hand side of (12) is negative. Likewise, the near-surface dissipative effects of friction reduce ζ_g near the surface, such that the left-hand side of (12) is more negative than were friction to be neglected. Because of the leading negative on the right-hand side of (12), and with h being positive, $\nabla^2\theta$ is positive such that θ is a local minimum.

We know, however, that a local minimum of θ at the surface is akin to an upper tropospheric negative (or anticyclonic) IPV anomaly. Like its upper tropospheric counterpart, this surface cold potential temperature anomaly induces a negative IPV anomaly upward toward the tropopause. Since the strength of this anomaly decreases with increasing height, it only slightly counteracts the positive IPV anomaly in the upper troposphere. Acting over a sufficiently long period of time, however, this process can result in the gradual decay of the positive IPV anomaly aloft. Such a process can be hastened in the presence of middle tropospheric diabatic warming which, as discussed in the context of the vertical diabatic term earlier in this lecture, results in the reduction of IPV above the level at which the warming is maximized.

Decay of an Upper Tropospheric Negative IPV Anomaly

Now, let us consider the decay of an upper tropospheric negative (or anticyclonic IPV) anomaly. This anomaly, by the "action at a distance" principle, induces a weak negative IPV anomaly at the surface that is eroded by friction. As geostrophic relative vorticity decreases (or becomes more anticyclonic) with increasing height, $\partial\zeta_g/\partial p$ is positive ($\partial\zeta_g < 0$, $\partial p < 0$). From thermal vorticity, this is associated with a warm surface potential temperature anomaly that we know is akin to an upper tropospheric positive IPV anomaly. As before, however, the weak positive IPV anomaly that this warm surface potential temperature anomaly induces in the upper troposphere is insufficient to completely weaken the upper tropospheric negative IPV anomaly. But, again, acting over a sufficiently long period of time, this process can result in the gradual decay of the negative IPV anomaly aloft.

Development of Lower to Middle Tropospheric Positive IPV Anomalies

In closing, let us return to our example of condensation and diabatic warming occurring above evaporation and diabatic cooling. Such a configuration is often found in the stratiform rain region of mesoscale convective systems (MCSs) as well as in the isentropic upglide region found along and to the north of a warm front. From our above discussion, we know that a diabatic warming maximum results in increased IPV below and decreased IPV above the height at which the warming is maximized. Conversely, a diabatic cooling maximum results in increased IPV above and decreased IPV below the height at which the cooling is maximized.

In between the diabatic warming and diabatic cooling maxima, we find that there is a constructive superposition of forcing for increased IPV! As we might expect, this overlap occurs in the lower-to-middle troposphere, particularly between 500-700 hPa. This configuration, if long-lasting enough, can result in the formation of middle tropospheric mesoscale convective vortices (MCVs) in the trailing stratiform region of an MCS. Likewise, if long-lasting enough, it can result in the formation of a lower-to-middle tropospheric diabatic Rossby vortex (DRV) along or immediately north of a warm frontal boundary. DRVs have been observed to occasionally result in explosive maritime synoptic-scale surface cyclogenesis, further highlighting the importance of diabatic heating to the surface cyclogenesis process.