Assignment #3: Effects of the Numerical Approximations

Part II: Numerical Dispersion and Numerical Diffusion

Due: 17 October 2017

Objective

Through practical experiments using a highly simplified numerical model, a) quantify the effects of numerical dispersion and b) demonstrate benefits and drawbacks from both implicit and explicit numerical diffusion to forecast quality and accuracy.

Introduction

This assignment uses the same numerical model formulation as in Assignment 2. Please refer to the description of that assignment for relevant instructions. All integrations are for 100,000 s.

Continued Exercises in Advecting a 1-D Gaussian Wave

1) (20 pts) Using a *second-order centered-in-time, second-order centered-in-space* finite difference scheme with a Courant number of 0.5, integrate your model forward. Repeat for a Courant number of 0.1. Create a plot with the two model solutions and that from Question 4 in Assignment 2 at only the final time.

    a. (7.5 pts) Describe how the final model solutions differ from each other.

    b. (12.5 pts) Fig. 3.25 of the course text depicts the wavelength and Courant number dependence of the phase and group velocities for this finite difference scheme. Given this information, describe why the final model solutions in part (a) differ.

2) (20 pts) Using a *second-order centered-in-time, second-order centered-in-space* finite difference scheme with a Courant number of 0.1, integrate your model forward twice, once each for a Gaussian wave with $c = 2$ and $c = 8$. Create a plot depicting the initial conditions ($t = 0$) and the model solutions at the final time for the cases where $c = 2$, 4, and 8.

    a. (7.5 pts) Describe how the final model solutions differ from each other.

    b. (12.5 pts) Given the initial wave structures and the information conveyed in Fig. 3.25 of the course textbook, describe why the final model solutions in part (a) differ.

3) (20 pts) Repeat Question 2, except for the following two waves:

$$h(x) = \frac{100}{b}\left(x - b\left\lfloor \frac{x}{b} + \frac{1}{2} \right\rfloor\right)\left(-1^{\left\lfloor \frac{x}{b} + \frac{1}{2} \right\rfloor}\right) \qquad \text{(triangle wave)}$$

If $h(x)$ from the triangle wave ≤ 0, $h(x) = -100.0$

(square wave)

If $h(x)$ from the triangle wave > 0, $h(x) = 100.0$

For the triangle wave, $\lfloor \ \rfloor$ indicates the "floor" function. Let $b = 10$ such that the triangle and square waves are periodic over the model domain.

a. (7.5 pts) Describe how the final model solutions differ from each other.

b. (12.5 pts) Given the initial wave structures and the information conveyed in Fig. 3.25 of the course textbook, describe why the final model solutions in part (a) differ.

In the first three questions, we demonstrated the deleterious effects of numerical dispersion on the modeled representation of various wave-like features. Now, we wish to demonstrate how implicit and explicit numerical diffusion can be used to mitigate these effects.

4) (20 pts) The 1-D advection equation underlying our model can be rewritten with a second-order explicit diffusion term as follows:

$$\frac{\partial h}{\partial t} = -U\frac{\partial h}{\partial x} + K\frac{\partial^2 h}{\partial x^2}$$

Here, K is a positive-definite diffusion coefficient. Using a *second-order centered-in-time, second-order centered-in-space* (for first **and** second partial derivatives) finite difference scheme, with $K = 3.0\text{x}10^3$ m^2 s^{-1}, a Courant number of 0.1, and the default Gaussian wave with $c = 4$, integrate your model forward. Create a plot with the model solution at the initial, second, final, and two intermediate times.

a. (10 pts) Describe how the model solution evolves with time. How does it compare to that from Question 1?

b. (10 pts) The scale selectivity of this second-order explicit diffusion term is depicted in Fig. 3.34 of the course text. Given this information, describe why the solutions in part (a) differ from those in Question 1.

5) (20 pts) Using a *Runge-Kutta 3, sixth-order centered-in-space* finite difference scheme and a Courant number of 0.1, integrate your model (without an explicit diffusion term) forward. Create a plot with the model solution at the initial, second, final, and two intermediate times. Describe how the model solution evolves with time. How does it compare to that from Question 1 for the centered-in-time, 2^{nd}-order centered-in-space difference scheme?