Assignment #2: Effects of the Numerical Approximations

Part I: Differencing Properties and Linear Stability

Due: 12 October 2017 (in class)

Objective

Through practical experiments using a highly simplified numerical model, quantify the effects of finite differencing schemes – particularly their accuracy, linear numerical stability, and implicit damping characteristics – upon forecast quality and accuracy.

Introduction

For this assignment and the next, you will conduct several experiments with a numerical model based upon a one-dimensional linear advection equation, i.e.,

$$\frac{\partial h}{\partial t} = -U \frac{\partial h}{\partial x}$$

Your model may be programmed, and its output visualized, in any programming language(s) of your choice. For those with at least some rudimentary experience with, or a willingness to learn, MATLAB or Python, you may find one of those to be ideal for this assignment. If you choose to use FORTRAN 77/90 and GrADS, both of which are also available on the Macs in W434, some helpful tips for doing so are provided later in this assignment.

As with all assignments for this class, while you are free to talk about this assignment with your classmates, all work that you complete and turn in must be your own.

Model Configuration

Some elements of your model will not change between individual experiments. These include:

- **Grid:** 100 grid points, with Δx = 10 km and periodic lateral boundary conditions (i.e., what goes out of the domain at one end enters at the other end).
- **Advective velocity:** 10 m s^{-1}.
- **Duration of integration:** 100,000 s (i.e., the time that it takes an initial feature to be advected over the length of the domain back to its initial location, such that the wave at the final time is identical to that at the start time).
- **Initial wave center point:** Grid point 50 (i.e., the center of the domain).

Other elements of your model may change between individual experiments.

Experiment: Advection of a 1-D Gaussian Wave

A 1-D Gaussian wave may be defined generically as:

$$h(x) = a \exp\left(\frac{-(x-b)^2}{2c^2} \right)$$

For this wave, a defines the initial amplitude of the wave, b defines the grid point where the initial wave is centered, and c is related to the initial width of the wave. For the following questions, a = 100, c = 4, and b is as defined earlier in the assignment.

1) (10 pts) For a Courant number of 0.8, given the advective velocity and model grid spacing stated previously, determine the necessary Δt and number of time steps for your model. Show all work. (Note that unless specified, you will use this time step in all subsequent questions to integrate your model.)

2) (20 pts) Using a *forward-in-time, backward-in-space* finite difference scheme, integrate your model forward for 100,000 s. Create a plot with the model solution at five times: the initial time, the second time, the final time, and two intermediate times of your choosing. Use a different line color and/or line style for each time. Ensure that each time is labeled appropriately, whether on the figure itself or in an appropriate figure caption. Describe how the amplitude of the Gaussian wave changes with time. What property of the chosen finite difference scheme results in this evolution?

3) (25 pts) Using a *forward-in-time, second-order centered-in-space* finite difference scheme, integrate your model forward for 100,000 s. Create a plot with the model solution at five times: the initial time, the second time, and time steps 20, 25, and 35.

    a. (7.5 pts) Describe how the model solution evolves with time.

    b. (17.5 pts) The stability criterion for this finite difference scheme is given by the equation at the bottom of p. 65 of the course textbook. Determine the resolved wavelength for which the exponential is maximized. Discuss the significance of this wavelength. Do you see evidence of this significance in your plot? Describe.

4) (20 pts) Using a *second-order centered-in-time, second-order centered-in-space* finite difference scheme, integrate your model forward for 100,000 s. Create a plot of the model solution at the same five times as in Question 2. How does the amplitude of the Gaussian wave change with time? How does this compare to the case in Question 2? Briefly describe any other differences that exist between these solutions.

5) (25 pts) Using a *second-order centered-in-time, second-order centered-in-space* finite difference scheme with a Courant number of 1.1, integrate your model forward for 100,000

s, making sure to change the time step and number of time steps accordingly. Create a plot with the model solution at five times: the initial time, the second time, and time steps 30, 35 and 40.

a. (4 pts) What are the Δt and number of time steps? Show all work.

b. (5 pts) Describe how the model solution evolves with time.

c. (8 pts) The stability criterion for this finite difference scheme is given by equation (3.54) of the course textbook. For a Courant number of 1.1, at what wavelength(s) is this finite difference scheme numerically unstable?

d. (8 pts) How does the solution differ from that in Question 3? In light of your answer to (c) and the wave's initial structure, discuss why these differences may arise.

Helpful Tips for FORTRAN 77/90 and GrADS

If your model data are stored in an array named **H** with dimensions 1→100 (X) in *x* and 1→# of time steps+1 (T) in *t*, then the following FORTRAN statements will write the entire array out to a new binary file named output.dat:

```
      OPEN(10,FILE='output.dat',FORM='unformatted',
c              ACCESS='direct',STATUS='new',RECL=4*X*T)

      WRITE(10,REC=1) H

      CLOSE(10)
```

The following text, saved to a file named something like output.ctl, comprises a GrADS control file. Control files provide metadata, or descriptions of the data found in a specified dataset, to allow GrADS to read in and visualize the data. Note that you may need to change the dset and tdef entries (bolded below, excluding the ^ in dset) if these change between model simulations.

dset ^**output.dat**
undef -9999.
title output from 1-d advection model
xdef 100 linear 1 1
ydef 1 linear 1 1
zdef 1 levels 1000
tdef **T+1** linear 00z01jan2000 1hr
vars 1
h 1 99 height (m)
endvars

I highly recommend that you write output from each model integration to a separate binary output file. You should also create a new GrADS control file, using the template above, to read the output from each model integration.

When visualizing data in GrADS, the default *x*-axis is a map-based longitude coordinate. To instead specify a non-map-based coordinate, type "set mproj off" (without the quotes) before displaying any data. I also recommend that you set your background color to white by typing "set display color white" and then "clear" (each without the quotes) immediately after opening GrADS.

If you wish to display data from a single file but at multiple times in GrADS,

set t 1
display h
set t 5

```
display h
set t 25
display h
```

and so on. If you wish to display data from multiple files at a single time in GrADS,

```
open output.ctl
open output2.ctl
open output3.ctl
set t 1
display h.1
display h.2
display h.3
```

where h.1 refers to variable h from output.ctl (the first file opened), h.2 refers to variable h from output2.ctl (the second file opened), and h.3 refers to variable h from output3.ctl (the third file opened). If you wish to display data from multiple files at different times in GrADS,

```
open output.ctl
open output2.ctl
open output3.ctl
display h.1(t=10)
display h.2(t=15)
display h.3(t=5)
```

where any time can be specified in the t=# statement in parentheses. Here, variable h in output.ctl at t=10, variable h in output2.ctl at t=15, and variable h in output3.ctl at t=5 will be displayed.

When you have an image that you wish to save to a file,

```
gxprint ~/output.png x1280 y1024
```

will save what is in your display window to a file named output.png (in your home directory) with dimensions 1280x1024. You can change the output file name as desired; e.g., to save the output from different questions' exercises to different files.