DART Tutorial Part II:
How should observations impact an unobserved state variable? Multivariate assimilation.
Single observed variable, single unobserved variable.

So far, have known observation likelihood for single variable.

Now, suppose prior has an additional variable.

Will examine how ensemble methods update additional variable.

Basic method generalizes to any number of additional variables.
Ensemble filters: Updating additional prior state variables

Assume that all we know is the prior joint distribution.

One variable is observed.

What should happen to the unobserved variable?
Ensemble filters: Updating additional prior state variables

Assume that all we know is the prior joint distribution.

One variable is observed.

Update observed variable with one of the previous methods.
Assume that all we know is the prior joint distribution.

One variable is observed.

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Update observed variable with one of the previous methods.
Assume that all we know is the prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.
Ensemble filters: Updating additional prior state variables

As we'll see, by computing the increments, we guarantee that if the observation doesn't impact the observed variable, the unobserved variable is unchanged.

This is highly desirable!
Ensemble filters: Updating additional prior state variables

Have joint prior distribution of two variables.

How should the unobserved variable be impacted?

1st choice: least squares

Begin by finding least squares fit.
Ensemble filters: Updating additional prior state variables

Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.
Ensemble filters: Updating additional prior state variables

Have joint prior distribution of two variables.

Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.
Ensemble filters: Updating additional prior state variables

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Now have an updated (posterior) ensemble for the unobserved variable.
Ensemble filters: Updating additional prior state variables

Now have an updated (posterior) ensemble for the unobserved variable.

Fitting Gaussians shows that mean and variance have changed.

Other features of the prior distribution may also have changed.
CRITICAL POINT:

Since impact on unobserved variable is simply a linear regression, can do this INDEPENDENTLY for any number of unobserved variables!

Could also do many at once using matrix algebra as in traditional Kalman Filter.
1. Use model to advance ensemble (3 members here) to time at which next observation becomes available.
2. Get prior ensemble sample of observation, $y = h(x)$, by applying forward operator $h$ to each ensemble member.

Theory: observations from instruments with uncorrelated errors can be done sequentially.
3. Get **observed value** and **observational error distribution** from observing system.
4. Find the **increments** for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).

Note: Difference between various ensemble filter methods is primarily in observation increment calculation.
5. Use ensemble samples of $y$ and each state variable to linearly regress observation increments onto state variable increments.

Theory: impact of observation increments on each state variable can be handled independently!
6. When all ensemble members for each state variable are updated, there is a new analysis. Integrate to time of next observation ...